

NON-ARCHIMEDEAN BIG PICARD THEOREMS

WILLIAM CHERRY

ABSTRACT. A non-Archimedean analog of the classical Big Picard Theorem, which says that a holomorphic map from the punctured disc to a Riemann surface of hyperbolic type extends across the puncture, is proven using Berkovich's theory of non-Archimedean analytic spaces.

1. INTRODUCTION

One of the fundamental theorems in complex analysis is a theorem of E. Picard that says an entire function must take on every complex value infinitely often, with at most one possible exception. Today, this is known as Picard's Little Theorem. A related theorem of Picard, today called Picard's Big Theorem, says that in a neighborhood of an isolated essential singularity, an analytic function must take on every value infinitely often, again with at most one possible exception. Picard's Big Theorem implies his little theorem because a non-polynomial entire function has an isolated essential singularity at the point at infinity on the Riemann sphere.

In fact, Picard also proved the following generalization of his little theorem to Riemann surfaces: Any holomorphic map from the complex plane to a Riemann surface of genus ≥ 2 must be constant, and any holomorphic map from the complex plane to a Riemann surface of genus 1 must hit every point in the curve infinitely often. An alternative way to phrase Picard's Big Theorem is to say that a function analytic on a punctured disc which omits at least two values must extend to a meromorphic function on the whole disc. Viewed this way, Picard's Big Theorem is an extension theorem, and in this form it can be generalized to analytic maps between Riemann surfaces. Recall that a Riemann surface is of hyperbolic type if its holomorphic universal covering space is biholomorphic to the unit disc. If a Riemann surface is not of hyperbolic type, then it is biholomorphic to one of the following: the Riemann sphere, the complex plane, the complex plane minus a point, or a complex torus. Picard proved the following:

Theorem 1.1 (Picard). *Let Y be a Riemann surface of hyperbolic type, let \bar{Y} be a compact Riemann surface containing Y , and let f be a holomorphic map from a punctured disc to Y . Then, f extends to a holomorphic map from the disc to \bar{Y} .*

In [Be], Berkovich proved a non-Archimedean analog of Picard's Little Theorem, namely any non-Archimedean analytic map from the affine line to a non-Archimedean Riemann surface of genus ≥ 1 must be constant. The purpose of this

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paper is to show how Berkovich’s theory can be used to prove a non-Archimedean analog of the Big Picard Theorem.

A corollary of Picard’s Big Theorem is that if $f : X \rightarrow Y$ is a holomorphic map between irreducible algebraic curves and if Y has genus ≥ 2 , then f must be a rational map. This corollary of the Big Picard Theorem was used in a fundamental way by Buium in [Bu] to give a new proof of the function field version of the following theorem of Raynaud [R]:

Theorem 1.2 (Raynaud). *Let $k \subset F$ be algebraically closed fields of characteristic zero. Let A be an Abelian variety defined over F with F/k trace zero. Let X be a projective algebraic curve in A with geometric genus ≥ 2 , and let Γ be a finite rank subgroup of $A(F)$. Then, $\Gamma \cap X(F)$ is finite.*

Raynaud did not make use of the Picard theorem. He used “reduction mod p^2 ” to reduce the theorem to the Mordell conjecture, which was proven in the characteristic zero function field case by Manin [M]. Buium’s observation that the Picard theorem could be used to prove Raynaud’s Theorem allowed him to generalize the theorem to higher dimensional X by using higher dimensional generalizations of Picard’s Big Theorem proved by Griffiths and King [G-K] and by Kobayashi and Ochiai [KO], which was the point of [Bu]. The theorem Buium proved had been a conjecture of S. Lang, see for instance [La1], [La2, Conjecture I.6.3]. E. Hrushovski [H] was able to prove Raynaud’s theorem and Buium’s higher dimensional generalization without the characteristic zero hypothesis by replacing Buium’s use of the Picard theorem with a model theoretic argument. To date, this remains the only proof valid in characteristic p . It would, however, be nice to have a proof of the characteristic p case of Lang’s Conjecture that does not involve model theory. One might try to generalize Buium’s approach to characteristic p , and the first step in doing so would be to prove a non-Archimedean version of the Big Picard Theorem, which is what is done in this paper.

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2. BASIC NOTIONS

Throughout this paper k will denote fixed algebraically closed field, complete with respect to a non-trivial non-Archimedean absolute value $|\cdot|$. The notation $|k|$ will be used to denote the set $\{|z| : z \in k\}$. Because k is algebraically closed and $|\cdot|$ is non-trivial, $|k|$ is dense in the non-negative real numbers. The quotient

$$\tilde{k} = \{z \in k : |z| \leq 1\} / \{z \in k : |z| < 1\}$$

is a field known as the **residue class field** of k .

By **analytic**, I will always mean analytic in the sense of Berkovich [Be]. Everything in this paper will be assumed to be defined over k . Furthermore, analytic spaces will always be assumed to be reduced and separated. Thus analytic space means a reduced, separated, Berkovich analytic space defined over k , and analytic map means a Berkovich analytic map defined over k .

The basic building blocks of non-Archimedean analytic spaces are “affinoid spaces.” The n -variable **Tate Algebra** over k , denoted $k\langle z_1, \dots, z_n \rangle$ is the Banach algebra

consisting of those formal power series $\sum_{\gamma} a_{\gamma} z^{\gamma}$, where the coefficients a_{γ} are in k and

$$\lim_{|\gamma| \rightarrow \infty} |a_{\gamma}| = 0.$$

Here γ is a multi-index. The Banach algebra norm on $k\langle z_1, \dots, z_n \rangle$ is the “sup-norm.” Namely,

$$\left| \sum_{\gamma} a_{\gamma} z^{\gamma} \right|_{\text{sup}} = \sup_{\gamma} |a_{\gamma}|.$$

The Berkovich analytic space associated to $k\langle z_1, \dots, z_n \rangle$ is referred to as the unit n -ball and denoted \mathbf{B}^n because the k -points of \mathbf{B}^n are those points (z_1, \dots, z_n) in k^n such that $\max\{|z_j| : 1 \leq j \leq n\} \leq 1$. The spaces \mathbf{B}^n play the role in non-Archimedean analytic geometry that affine spaces play in algebraic geometry. If a Banach algebra A defined over k is isomorphic to the quotient of a Tate algebra by a closed ideal, then A is called an **affinoid algebra**. The Berkovich analytic space associated to such an algebra is called an **affinoid space**. Note that in Berkovich’s terminology these would be called “strictly” affinoid.

Any algebraic variety X over k can be made into a Berkovich analytic space in a natural way. I use X , \mathbf{P}^1 , \mathbf{A}^1 , *etc.* to denote the analytic space associated to a variety X , the projective line, the affine line, *etc.* The k -points of these spaces will be denoted $X(k)$, $\mathbf{P}^1(k)$, $\mathbf{A}^1(k)$, *etc.*

Given $r > 0$, the “closed disc” of radius r and “center” 0 will be denoted $\mathbf{D}(r)$, which is the the affinoid analytic space associated to the Tate algebra

$$k\langle r^{-1}z \rangle = \left\{ \sum_{n=0}^{\infty} a_n z^n : \lim_{n \rightarrow \infty} |a_n| r^n = 0 \right\},$$

which is also the ring of analytic functions on $\mathbf{D}(r)$. The notation \mathbf{D} will stand for $\mathbf{D}(1)$. When $r \in |k|$, $\mathbf{D}(r)$ and \mathbf{D} are clearly isomorphic over k . The notation $\mathbf{D}^{\times}(r)$ will be used to denote the “punctured” disc of radius r , which is not affinoid, but whose analytic functions are of the form

$$\sum_{n=-\infty}^{\infty} a_n z^n, \quad \text{with } \lim_{n \rightarrow \infty} |a_n| r^n = 0 \text{ and } \lim_{n \rightarrow -\infty} |a_n| t^n = 0 \text{ for all } t > 0.$$

Occasionally I will need to refer to a disc with non-zero “center,” so $\mathbf{D}(a, r)$ will be used to denote the “closed disc” of radius r and “center” a .

Theorem 2.1 (Big Picard). *Let f be a meromorphic function on $\mathbf{D}^{\times}(r)$ which omits at least two values in $\mathbf{P}^1(k)$. Then, f extends to an analytic map from $\mathbf{D}(r)$ to \mathbf{P}^1 .*

Proof. By shrinking r a little bit if necessary, we may assume r is in $|k|$. By post-composition with a Möbius transformation, it suffices to assume that f omits the values 0 and ∞ . Thus, we may assume f is an analytic function on $\mathbf{D}^{\times}(r)$ and can be written

$$f = \sum_{n=-\infty}^{\infty} a_n z^n, \quad \text{with } \lim_{n \rightarrow \infty} |a_n| r^n = 0 \text{ and } \lim_{n \rightarrow -\infty} |a_n| t^n = 0 \text{ for all } t > 0.$$

For $0 < t \leq r$, let

$$|f|_t = \sup_n \{|a_n| t^n\} = \sup\{|f(z)| : z \in k \text{ with } |z| = t\}.$$

The last equality is the non-Archimedean maximum modulus principle – see [B-G-R, Proposition 5.1.3]. The valuation polygon of f is the graph of $\log |f|_t$ as a function of $\log t$. The valuation polygon is a piecewise linear graph with corners at those points $(\log t_0, \log |f|_{t_0})$ where there exist two integers $n_1 \neq n_2$ such that

$$|a_{n_1}|t_0^{n_1} = |f|_{t_0} = |a_{n_2}|t_0^{n_2}.$$

The theory of Newton or valuation polygons, which essentially amounts to Hensel’s Lemma (or the p -adic Weierstrass Preparation Theorem) and Gauss’s Lemma, says that for each corner $(\log t_0, \log |f|_{t_0})$ of the valuation polygon, f has a zero z_0 with $|z_0| = t_0$ – see [A, Ch. 4]. In fact, the number of zeros, counting multiplicity, is determined by the “sharpness” of the corner.

If there are infinitely many negative integers n with $a_n \neq 0$, then it is clear that the valuation polygon for f has infinitely many corners and hence that f has infinitely many zeros. Since f is assumed to be zero free, there can be at most finitely many negative n with $a_n \neq 0$, and hence f has at worst a pole at $z = 0$. Thus, f extends to a holomorphic map from $\mathbf{D}(r)$ to \mathbf{P}^1 . \square

Theorem 2.2 (Open Mapping). *Let $r \in |k|$ and let f be a non-constant analytic function on $\mathbf{D}(r)$. Then there exists a $\delta > 0$ and in $|k|$ such that the image of $\mathbf{D}(r)$ under f is precisely $\mathbf{D}(f(0), \delta)$.*

Proof. We first consider the k -points. Write

$$f(z) - f(0) = \sum_{n=m}^{\infty} a_n z^n,$$

with $m \geq 1$ and $a_m \neq 0$. Then for any w in k ,

$$f(z) - w = f(0) - w + \sum_{n=m}^{\infty} a_n z^n.$$

From the theory of valuation (Newton) polygons, we see that the series

$$f(0) - w + \sum_{n=m}^{\infty} a_n z^n$$

will have a zero in $\mathbf{D}(r)(k)$ if and only if

$$|f(0) - w| \leq \sup_{n \geq m} |a_n| r^n.$$

Hence, if we let

$$\delta = \sup_{n \geq m} |a_n| r^n, \quad \text{then} \quad f(\mathbf{D}(r)(k)) = \mathbf{D}(f(0), \delta)(k).$$

By [Be, Proposition 2.1.15], $\mathbf{D}(r)(k)$ is dense in $\mathbf{D}(r)$ and $\mathbf{D}(f(0), \delta)(k)$ is dense in $\mathbf{D}(f(0), \delta)$. Because $\mathbf{D}(r)$ and $\mathbf{D}(f(0), \delta)$ are compact (see [Be, Proposition 1.2.3]) and f is continuous, the image of $\mathbf{D}(r)$ under f must be $\mathbf{D}(f(0), \delta)$. \square

The reason that I prefer to work with Berkovich’s notion of analytic spaces is that they are rather nice geometric spaces, unlike the more traditional “rigid analytic” spaces. For example, we have the following basic geometric fact.

Theorem 2.3. *The Berkovich analytic space $\mathbf{D}^\times(r)$ is arc-connected and simply connected.*

Proof. See [Be, e.g., Theorem 4.2.1]. \square

The following lemma concludes this section.

Lemma 2.4. *Let X and Y be connected one dimensional non-singular analytic spaces, defined over k . Let $h: X \rightarrow Y$ be a non-constant analytic map with finite fibers defined over k , and let $f: \mathbf{D}^\times(r) \rightarrow X$ be an analytic map defined over k such that $h \circ f$ extends to an analytic map from $\mathbf{D}(r)$ to Y . Then, f itself extends to an analytic map from $\mathbf{D}(r)$ to X .*

Proof. Let $y_0 = h(f(0))$, which is a point in $Y(k)$. Let x_j be the points in $h^{-1}(y_0)$, which form a finite set of points in $X(k)$. We can find an affinoid neighborhood V of y_0 and affinoid neighborhoods U_j of each point x_j and each isomorphic to \mathbf{D} such that that $U_i \cap U_j$ is empty for all $i \neq j$, and such that for each j , $h(U_j)$ is contained in V . Because $h(f(0)) = y_0$, we have $h(f(\mathbf{D}(\varepsilon)))$ contained in V , for ε sufficiently small. Because $\mathbf{D}^\times(\varepsilon)$ is connected by Theorem 2.3, this implies that $f(\mathbf{D}^\times(\varepsilon))$ is contained in just one of the sets U_j . Because U_j is isomorphic to \mathbf{D} , we can apply Theorem 2.1 to conclude that f extends to an analytic map from $\mathbf{D}(\varepsilon)$ to U_j , and hence f extends to an analytic map from $\mathbf{D}(r)$ to X . \square

3. REDUCTION AND UNIFORMIZATION OF CURVES

Let Γ be a subgroup of $PGL(2, k)$. A point z in \mathbf{P}^1 is called a **limit point** of Γ if there exist a point w in \mathbf{P}^1 and an infinite sequence γ_n in Γ such that $\lim \gamma_n(w) = z$. We let Σ_Γ denote the set of limit points of Γ . The group Γ is called **discontinuous** if $\Sigma_\Gamma \neq \mathbf{P}^1$ and if for each z in \mathbf{P}^1 , the orbit of z under Γ is compact. The group Γ is called a **Schottky group** if it is finitely generated, discontinuous, and contains no non-trivial torsion elements.

Recall that a set is called **perfect** if it is equal to its set of limit points. In other words a perfect set is a closed set which does not contain any isolated points.

Theorem 3.1. *Let Γ be a Schottky group of rank $g \geq 1$ with limit set Σ_Γ . Then, Σ_Γ is contained in $\mathbf{P}^1(k)$. Let $\Omega = \mathbf{P}^1 \setminus \Sigma_\Gamma$. Then, Γ acts freely on Ω , the quotient space $X = \Omega/\Gamma$ is a smooth projective algebraic curve of genus g , and the natural map $\pi: \Omega \rightarrow X$ is an analytic universal covering map. Moreover, if $g = 1$, then Σ_Γ consists of exactly two points and if $g \geq 2$, then Σ_Γ is a perfect set.*

Proof. See [G-P] and [Be, §4.4]. \square

A curve X which is the quotient of a Schottky group as in Theorem 3.1 is called a **Mumford curve**.

Given an analytic space, it is possible to associate to it an algebraic variety \tilde{X} defined over the residue class field \tilde{k} . The variety \tilde{X} is called a **reduction** of X and need not be unique. I will now recall this notion.

In the case that X is affinoid, there is a canonical reduction \tilde{X} . To describe this more fully, recall that any affinoid algebra A has a semi-norm known as the sup-semi-norm and denoted $|\cdot|_{\text{sup}}$.

Thus, one defines

$$\tilde{A} = \{f \in A : |f|_{\text{sup}} \leq 1\} / \{f \in A : |f|_{\text{sup}} < 1\},$$

which is an algebra over \tilde{k} . In fact, since A is affinoid and therefore a quotient of a Tate algebra by a closed ideal, \tilde{A} is the quotient of a polynomial ring over \tilde{k} by an ideal, and hence $\tilde{X} = \text{Spec } \tilde{A}$ is an affine algebraic variety of \tilde{k} . For example, the reduction of the n -ball \mathbf{B}^n is the affine space \mathbf{A}^n over \tilde{k} . Affinoid analytic spaces

also have a canonical reduction map $\pi: X \rightarrow \tilde{X}$. See [Be, §1.3] or [B-G-R, §6.2] for more details on the supremum semi-norm, the reduction of affinoid spaces, and the reduction map π .

Before discussing the reduction of more general analytic spaces, I must recall the notion of formal admissible affinoid coverings. Let V be an affinoid subspace of an affinoid space U . The inclusion of V into U induces a morphism from \tilde{V} to \tilde{U} . If the induced morphism on the reductions is an open immersion, then V is called a **formal** affinoid subdomain of U . Recall that by an **admissible affinoid cover** \mathcal{U} of an analytic space X , one means a cover \mathcal{U} consisting of affinoid subspaces U such that if V is any affinoid subspace of X , then $\mathcal{U}|_V = \{U \cap V\}$ is a *finite* covering of V . An admissible affinoid covering \mathcal{U} is called **formal** if $U \cap V$ is a formal affinoid subdomain of U for every pair U, V in \mathcal{U} . Given a formal admissible affinoid covering \mathcal{U} of X , one gets an algebraic variety $\tilde{X}_{\mathcal{U}}$ defined over \tilde{k} and a reduction map $\pi: X \rightarrow \tilde{X}_{\mathcal{U}}$. In general, different coverings may give rise to non-isomorphic reductions.

In general, all reductions $\tilde{X}_{\mathcal{U}}$ may be singular varieties even if X is a nice non-singular space. If $\tilde{X}_{\mathcal{U}}$ is non-singular, then X is said to have **good reduction**. Fortunately in the case of curves, one has some control over how bad the singularities of $\tilde{X}_{\mathcal{U}}$ can be.

Theorem 3.2 (Semi-Stable Reduction). *If X is a non-singular connected projective algebraic curve, then there exists a formal admissible affinoid covering \mathcal{U} of X so that $\tilde{X}_{\mathcal{U}}$ is reduced with at worst ordinary double point singularities, and*

- (a) *If X has genus 0 then $\tilde{X}_{\mathcal{U}}$ is isomorphic to \mathbf{P}^1 .*
- (b) *If X has genus 1, then every non-singular rational component of $\tilde{X}_{\mathcal{U}}$ meets the other components in at least two points. (Such a reduction is called “semi-stable.”) Moreover, if $\tilde{X}_{\mathcal{U}}$ is non-singular, then up to isomorphism, there exists at most one semi-stable reduction of X .*
- (c) *If X has genus ≥ 2 , then every non-singular rational component of $\tilde{X}_{\mathcal{U}}$ meets the other components in at least three points. (Such a reduction is called “stable.”) Moreover, up to isomorphism, there exists at most one stable reduction of X .*

Moreover, $\dim H^1(\tilde{X}_{\mathcal{U}}, \mathcal{O}_{\tilde{X}_{\mathcal{U}}}) = \dim H^1(X, \mathcal{O}_X) = g$.

Proof. See [B-L]. □

Note that if X has genus ≥ 1 and $\tilde{X}_{\mathcal{U}}$ is a reduction of X assured by Theorem 3.2, then either every component of $\tilde{X}_{\mathcal{U}}$ is rational or $\tilde{X}_{\mathcal{U}}$ has at least one non-rational component. This dichotomy does not depend on the covering \mathcal{U} because if X has genus 1, the only case where the specified reduction may not be unique, and if $\tilde{X}_{\mathcal{U}}$ is singular, then every component must be rational. A curve of genus ≥ 1 is said to have **totally degenerate reduction** if every component of a reduction $\tilde{X}_{\mathcal{U}}$ as in Theorem 3.2 is rational. The reduction of curves with totally degenerate reduction is not much use in studying Picard type questions for maps into X . Fortunately it is precisely curves with totally degenerate reduction that have a nice uniformization theory, and that uniformization theory can be used to prove Picard type theorems in the totally degenerate reduction case.

Theorem 3.3. *Let X be a non-singular projective algebraic curve of genus $g \geq 1$. Then, X is a Mumford curve if and only if X has totally degenerate reduction.*

Proof. See [Be, Theorem 4.4.1]. \square

Finally, we recall that analytic maps lift to normalizations.

Theorem 3.4. *Let $f: X \rightarrow Y$ be an analytic map between reduced analytic spaces. Let $N(Y)$ denote the analytic subvariety of non-normal points in Y . Assume that $f^{-1}(N(Y))$ is nowhere dense in X . Let $\pi_X: \widehat{X} \rightarrow X$ and $\pi_Y: \widehat{Y} \rightarrow Y$ be the normalizations of X and Y . Then, there exists a unique analytic map \widehat{f} from \widehat{X} to \widehat{Y} such that $\pi_Y \circ \widehat{f} = f \circ \pi_X$.*

Proof. The classical complex analytic argument [G-R] goes through. See [Be, §3.3] and [Lü] for some details. \square

4. NON-ARCHIMEDEAN ANALYTIC MAPS TO ALGEBRAIC CURVES

Theorem 4.1. *Let f be an analytic map from $\mathbf{D}^\times(r)$ to an irreducible projective algebraic curve X . Suppose one of the following: (a) X has geometric genus ≥ 2 ; (b) X has geometric genus 1 and either f omits at least one point of X or the normalization of X has good reduction; or (c) X has geometric genus 0 and f omits at least two points of X . Then f extends to an analytic map from $\mathbf{D}(r)$ to X .*

Proof. By Theorem 3.4, it suffices to prove the theorem when X is smooth, and the case when X has genus 0 is already covered by Theorem 2.1.

When X has positive genus, the proof breaks up into two cases.

Case 1. Suppose X is a Mumford curve. Then, by Theorem 3.1, there exists a closed set Σ in $\mathbf{P}^1(k)$ and a universal covering map π from $\Omega = \mathbf{P}^1 \setminus \Sigma$ to X . By Theorem 2.3, f lifts to a map \widehat{f} from $\mathbf{D}^\times(r)$ to Ω .

If X has genus ≥ 2 , then Σ is a perfect subset of $\mathbf{P}^1(k)$ by Theorem 3.1. Since Σ contains more than two points, Theorem 2.1 tells us that \widehat{f} extends to an analytic map from $\mathbf{D}(r)$ to \mathbf{P}^1 . By Theorem 2.2, there exists $\varepsilon > 0$, such that $\mathbf{D}(f(0), \varepsilon)$ is contained in the image of \widehat{f} . Because Σ is perfect, this implies $f(0)$ cannot be in Σ , and hence \widehat{f} maps to Ω . Thus, $\pi \circ \widehat{f}$ is an analytic map from $\mathbf{D}(r)$ to X which extends f .

If X has genus 1, then by assumption f omits a point x_0 of X . Moreover, X is a Tate curve, so we may assume Σ consists of the points 0 and ∞ and that π is the quotient of the multiplicative group modulo a rank 1 multiplicative subgroup. Thus, $\pi^{-1}(x_0)$ accumulates at both 0 and ∞ . Because \widehat{f} omits the two points in Σ , it extends to a map from $\mathbf{D}(r)$ to \mathbf{P}^1 by Theorem 2.1. If $f(0)$ is not zero or infinity, then $\pi \circ \widehat{f}$ extends f to a map from $\mathbf{D}(r)$ to X and we are done. But, $f(0)$ cannot be zero or infinity, for in that case Theorem 2.2 would imply that the image of f contained infinitely many of the points in $\pi^{-1}(x_0)$, a contradiction.

Case 2. If X is not a Mumford curve, then by Theorem 3.3, X has a formal affinoid covering \mathcal{U} such that $\widehat{X_{\mathcal{U}}}$ contains an irreducible component with positive geometric genus. As in Berkovich's proof of the little Picard Theorem, [Be, Theorem 4.5.1], there exists a rational function h from X to \mathbf{P}^1 , such that $h \circ f$ is bounded in \mathbf{A}^1 . Applying Theorem 2.1, we see that $h \circ f$ extends to an analytic map from $\mathbf{D}(r)$ to \mathbf{P}^1 , and then applying Lemma 2.4, we see that f itself extends to an analytic map from $\mathbf{D}(r)$ to X . \square

Corollary 4.2. *Let $f: X \rightarrow Y$ be an analytic map between smooth irreducible algebraic curves defined over k . If Y has geometric genus ≥ 2 , then f is a rational map.*

Proof. Let \overline{X} and \overline{Y} be smooth projective curves containing X and Y as Zariski open subsets. Then, $\overline{X} \setminus X$ is a finite set of isolated points. The mapping f extends to an analytic map in a neighborhood of each point in $\overline{X} \setminus X$, by Theorem 4.1. Thus, f extends to an analytic map from \overline{X} to \overline{Y} . By the GAGA Theorem, [Be, §3.4], f is rational. \square

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5. EPILOG: JULY 9, 2002

I submitted this paper to the *Proceedings of the American Mathematical Society* in August 2000. There was a considerable delay in getting the manuscript refereed, which might have been partially my fault. Once finally refereed, the referee felt that Theorem 4.1, the original content of this paper, is a “relatively simple consequence of the (deep) semistable reduction theorem quoted as Theorem 3.2.” I do not disagree, but unlike the referee, I thought a write up of the details of how that argument goes, as well as my description of a potential application, might be a useful contribution to the literature. Because I am currently working on a manuscript which will be much more general than what is here, I have decided not to try to publish this paper elsewhere, but will keep the paper available on my web page and submit it to a preprint archive.

One other comment is in order. Theorem 2.1 can be found in:

M. VAN DER PUT, *Essential singularities of rigid analytic functions*, *Nederl. Akad. Wetensch. Indag. Math.* **43** (1981), 423–429.
MR **83h**:14021

I always suspected I was not the first to make this observation, and shortly after I submitted my paper I found the above reference. The referee was also kind enough to point the above reference out to me. As also noted by the referee, Theorem 2.1 can now also be found in the wonderful book:

A. ROBERT, *A course in p -adic analysis*, *Graduate Texts in Mathematics* **198**, Springer-Verlag, 2000. MR **2001g**:11182

DEPARTMENT OF MATHEMATICS, P.O. BOX 311430, UNIVERSITY OF NORTH TEXAS, DENTON,
TX 76203

E-mail address: wcherry@unt.edu