Errata for *Nevanlinna's Theory of Value Distribution. The Second Main Theorem and Its Error Terms,* by William Cherry and Zhuan Ye, Published by Springer-Verlag, 2001

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Errata

The following describe various errors. Those errors where the page number is in bold face type are the most likely to lead to confusion. Negative line numbers count from the bottom of the page.

Page	**Line(s)**	**Correction**
vi	8–10	Both . . . have . . .
6	-8	“being” $\mapsto$ “begin”
8	-2	$\phi(Re^{i\theta}) \mapsto \phi(Re^{i\theta_0})$ on the left side of the $=$.
14	-7	“the origin” $\mapsto (0, 0, 1/2)$
14	-4	$\zeta_j = \dfrac{r_j^2 - 1}{4(1 + r_j^2)} \mapsto \zeta_j = \dfrac{r_j^2}{1 + r_j^2}$
16	3–5	Should only be stated for $a = \infty$, *i.e,* $$m(f, \infty, r) \le \mathring{m}(f, \infty, r) \le m(f, \infty, r) + \frac{\log 2}{2}$$
34	3	“three” $\mapsto$ “four”
34	-9	**GJ 4** $\mapsto$ **GJ 2**
53	3	“$\xi = \phi$” $\mapsto$ “$\xi = \psi$”
78	-4	“those reader” $\mapsto$ “those readers”
61	-14	Insert comma before “in §7.1”
86	1–2	The surface X needs to be better explained.
88	15	“$J_N =$ ” $\mapsto$ “$J_N \approx$ ”
108	-12	“beginning of the section” $\mapsto$ “beginning of the chapter”
113	17	$$\sum_{\substack{j=1 \\ j \neq j_0}}^{q'} \log^+ \lvert a_j \rvert \quad \mapsto \quad \max_{\substack{1 \le j \le q' \\ j \neq j_0}} \log^+ \lvert a_j \rvert$$
120	-5	“polynomial” $\mapsto$ “polynomial P_q”
122	-1	Too much space before the comma.
134		The proof given is not correct if P has zero constant term. One needs to consider the more general change of variables, $g = 1/f + c$ for a constant c.
135	21	“is constant also” $\mapsto$ “is also constant”
135	-1	$N_{\mathrm{ram}}(f, r) \mapsto N_{\mathrm{ram}}(f_\ell, r)$
157	21–23	The “Roth type conjecture” cannot be true with the strong error-term stated here. See below.

Number theoretic Error Terms

The "Roth type conjecture" with truncated counting functions listed in the right hand side of the table on page 157 cannot be true with the strong error term, as stated. This sheds some doubt onto the strength of the connection between the error terms in Nevanlinna theory and in Diophantine approximation.

If the Roth type conjecture on page 157 were true, then with the notation as on page 157, we would have

$$h(x) \leq N^1(\mathbf{Q}, 0, x) + N^1(\mathbf{Q}, \infty, x) + N^1(\mathbf{Q}, 1, x) + \log h(x) + \log \psi(h(x)) + O(1)$$

for all rational numbers $x \in \mathbf{Q}$, and for any Khinchin function ψ. Now suppose a_j, b_j, and c_j are relatively prime integers with $a_j + b_j = c_j$. Then, set $x_j = a_j/c_j$ and $y_j = b_j/c_j$. We would have from the Roth type conjecture that

$$\begin{aligned} h(x_j) \quad \leq \quad & N^1(\mathbf{Q}, 0, x_j) + N^1(\mathbf{Q}, \infty, x_j) + N^1(\mathbf{Q}, 1, x_j) \\ & + \log h(x_j) + \log \psi(h(x_j)) + O(1). \end{aligned}$$

Since a_j, and b_j are relatively prime, we also know

$$N^1(\mathbf{Q}, 0, x_j) + N^1(\mathbf{Q}, \infty, x_j) + N^1(\mathbf{Q}, 1, x_j) = N^1(\mathbf{Q}, 0, a_j b_j c_j).$$

Similarly,

$$h(y_j) \leq N^1(\mathbf{Q}, 0, a_j b_j c_j) + \log h(y_j) + \log \psi(h(y_j)) + O(1).$$

Define $h((a_j, b_j, c_j)) = \max\{h(x_j), h(y_j)\}$. We then have

$$h((a_j, b_j, c_j)) \leq N^1(\mathbf{Q}, 0, a_j b_j c_j) + \log h((a_j, b_j, c_j)) + \log \psi(h((a_j, b_j, c_j))) + O(1),$$

which is a strong form of the abc-Conjecture. So strong in fact, that it is false, as shown in a 1986 paper,

> C. L. Stewart and R. Tijdeman, *On the Oesterlé-Masser Conjecture,* Monatsh. Math. **102** (1986), 251–257,

where Stewart and Tijdeman construct infinitely many relatively prime triples of integers (a_j, b_j, c_j), such that $a_j + b_j = c_j$, but such that

$$h((a_j, b_j, c_j)) \geq N^1(\mathbf{Q}, 0, a_j b_j c_j) + C\sqrt{\frac{h((a_j, b_j, c_j))}{\log h((a_j, b_j, c_j))}}.$$

Thus, the Roth type conjecture is false for some choices of Khinchin function ψ, for example $\psi(x) = (\log x)^2$. See also the more recent work of van Frankenhuysen

> M. van Frankenhuysen, *A lower bound in the abc conjecture,* J. Number Theory **82** (2000), 91–95,

for an improvement in the constant C.

Note that the abc examples of Stewart and Tijdeman and of van Frankenhuysen constrain the best possible error term for a Roth type conjecture with truncated counting functions, but they do not provide counterexamples to Conjecture 6.2.1 on page 159. To date, there are no known counterexamples to Conjecture 6.2.1 with the strong error term as stated.