

Math 4060 (Cherry) Homework Assignment #31

Due Friday, April 28

Note: This is not a worksheet. Your solutions to the problems below should be on blank paper.

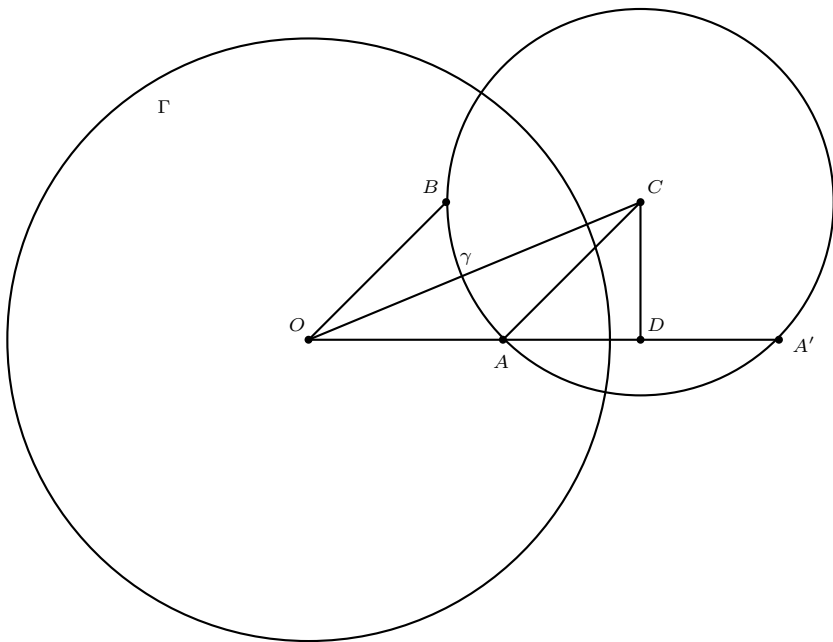
Studying geometry by using coordinates, algebra, trigonometry, calculus, and so forth is called “analytic geometry.” This assignment will show how we can study the Poincaré plane using analytic geometry.

Throughout this assignment we will fix O to be the point $(0, 0)$ in \mathbf{R}^2 and we will fix Γ to be the unit circle with center O , i.e.,

$$\Gamma = \{(x, y) \in \mathbf{R}^2 : x^2 + y^2 = 1\}.$$

The Poincaré plane will be all the points in \mathbf{R}^2 inside Γ .

1. Let $P = (1/2, 0)$ and $Q = (3/4, 0)$. Find the coordinates of the P-center of the P-circle with diameter \overline{PQ} .
2. In this problem, we will find the coordinates of the vertices of an equilateral triangle in the P-plane so that each angle of the triangle measures 45° . We will choose O as one of the vertices. We will let one of the vertices be a point A with coordinates $(a, 0)$, and we will let B be the third vertex. Our goal is to find the value of a .



- (a) If we want $\angle BOA$ to be 45° with $\overline{BO} \cong \overline{AO}$, then explain why the coordinates of B will be $(a/\sqrt{2}, a/\sqrt{2})$.
- (b) Find the coordinates of $A' = \rho_\Gamma(A)$.

Let γ be the P-line connecting A and B . Since γ is an E-circle, it will have an equation of the form

$$(x - h)^2 + (y - k)^2 = r^2.$$

Here $C = (h, k)$ is the E-center of γ and r is its E-radius. We will now find h , k , and r in terms of a .

- (c) Let D be the E-midpoint of E-segment $\overline{AA'}$. Find the coordinates of D . Since $\overline{AA'}$ is an E-chord of the E-circle γ , the perpendicular bisector of $\overline{AA'}$ passes through the E-center of γ . That means h is equal to the x -coordinate of D .
- (d) Consider the E-triangle $\triangle ACD$. You can determine each of the angles in this triangle and then use that to determine k . *Hint:* You should find that $k = (1 - a^2)/(2a)$.
- (e) Now that you know h and k you can find r in terms of a by plugging in the coordinates of A for x and y . In other words,

$$r^2 = (a - h)^2 + (0 - k)^2.$$

- (f) Now that you have r , h , and k all in terms of a , you can find an equation for a by plugging in the coordinates of B . What is the value of a ?