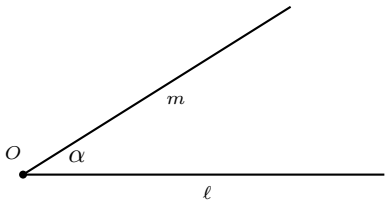
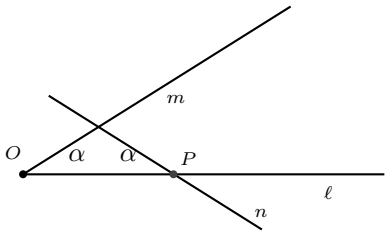
Math 4060 (Cherry) Homework Assignment #29
Due Monday, April 24

1. In this exercise, we will show that if α is any angle less than 60°, then there is an α-α-α-equilateral-triangle in the Poincaré plane.

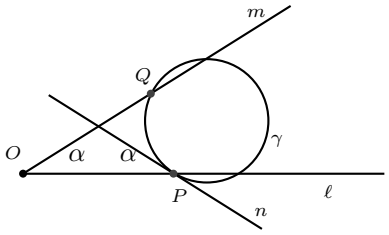
   (a) Begin with two lines ℓ and m meeting a point O and making an angle α.



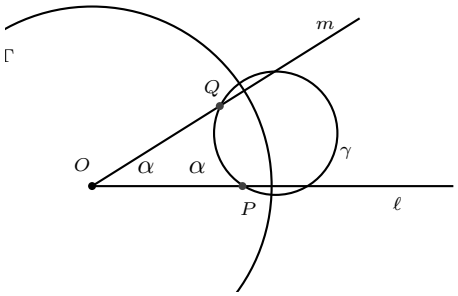
   (b) Now copy the Euclidean angle α to a point P on ℓ making a line n.



   (c) Explain why the fact that $\alpha < 60^\circ$ allows you to find an E-circle γ tangent to n at P and whose center lies on the bisector of the angle formed by ℓ and m. Then explain why this circle intersects m and let Q be the intersection point closest to O.



(d) Explain how to construct an E-circle Γ with center O and perpendicular to γ. [You do not need to give a step-by-step ruler & compass construction, but explain how you would find Γ.]



(e) Now, we will think of Γ as the circle defining the P-plane. Explain why OPQ is an equilateral P-triangle.