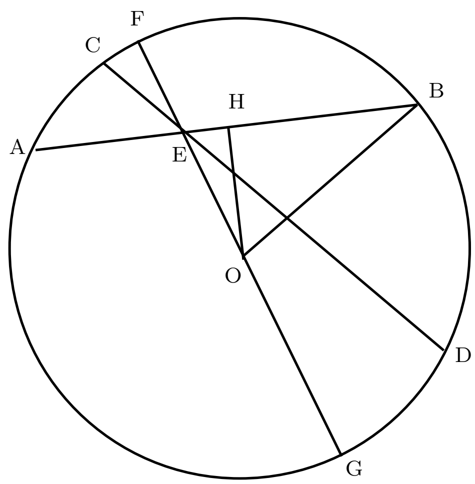

Euclid's proposition III.35 says that if $\overline{AB}$ and $\overline{CD}$ are chords of a circle intersecting in a point E, then $\overline{AE} \cdot \overline{EB} = \overline{CE} \cdot \overline{ED}$.

Let O be the center of the circle, and let $\overline{FG}$ be the diameter passing through E. Then, a special case of the proposition is to show that $\overline{AE} \cdot \overline{EB} = \overline{FE} \cdot \overline{EG}$. In fact, it is enough to show this special case, because if $\overline{AE} \cdot \overline{EB} = \overline{FE} \cdot \overline{EG}$ and if $\overline{CE} \cdot \overline{ED} = \overline{FE} \cdot \overline{EG}$, then by transitivity, $\overline{AE} \cdot \overline{EB} = \overline{CE} \cdot \overline{ED}$.

Hence, we will proceed to prove that $\overline{AE} \cdot \overline{EB} = \overline{FE} \cdot \overline{EG}$. Let r be the radius of the circle. To ease our notation, let $x = \overline{AE}$, let $y = \overline{EB}$, and let $z = \overline{FE}$. Then $\overline{EG} = 2r - z$ since the length of the diameter FG is $2r$.

Drop a perpendicular $\overline{OH}$ from O to $\overline{AB}$. Then by III.3, H is the midpoint of $\overline{AB}$. That means $\overline{BH} = (x+y)/2$. Let $h = \overline{OH}$. Applying the Pythagorean Theorem to the right triangle $\triangle OHB$ and noting that $\overline{OB}$ is a radius, we see that

$$h^2 = r^2 - \left(\frac{x+y}{2}\right)^2.$$

Now note that

$$\overline{EH} = \overline{EB} - \overline{HB} = y - \frac{x+y}{2} = \frac{y-x}{2}.$$

Also, $\overline{OE} = r - z$. So, if we now apply the Pythagorean Theorem to right triangle $\triangle OHE$, we have

$$h^2 = (r-z)^2 - \left(\frac{y-x}{2}\right)^2.$$

Combining the two equations for h^2, we then find

$$r^2 - \left(\frac{x+y}{2}\right)^2 = h^2 = (r-z)^2 - \left(\frac{y-x}{2}\right)^2.$$

Multiplying the left-hand and right-hand sides out, we get

$$r^2 - \frac{x^2}{4} - \frac{xy}{2} - \frac{y^2}{4} = r^2 - 2rz + z^2 - \frac{y^2}{4} + \frac{xy}{2} - \frac{x^2}{4}.$$

The r^2, x^2, and y^2 terms cancel from each side leaving

$$-\frac{xy}{2} = -2rz + z^2 + \frac{xy}{2}.$$

Now, we move x and y to the right and r and z to the left to get

$$2rz - z^2 = xy.$$

Factor a z out on the left, and finally we have

$$z(2r - z) = xy.$$

But this is exactly

$$\overline{FE} \cdot \overline{EG} = \overline{AE} \cdot \overline{EB},$$

which is precisely what we wanted.